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COST-BENEFIT RULES FOR PUBLIC GOOD PROVISION WITH DISTORTIONARY TAXATION

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Abstract

The paper shows that a comparison of the appropriately-weighted sum of households' marginal willingness to pay for a public good with the net effect of the increased supply of the public good on shadow, as distinct from actual, government revenue is a generally valid rule for public good provision. This rule does not depend on any assumption that existing policy is optimal. The practical problems in measuring the true social cost of additional public good provision involve the need to estimate shadow prices of non-traded goods and goods which are not traded at given world prices The marginal cost of public funds is not required in order to measure the social cost of public good provision.

JEL Classification: H2, H4.

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1 Introduction

The appropriate cost-benefit rule for determining whether a marginal increase in public good provision is desirable in an economy using distortionary taxation is the subject of a substantial literature (see, among others, Diamond and Mirrlees 1971, Stiglitz and Dasgupta 1971, Atkinson and Stern 1974, Wildasin 1984, King 1986, Wilson 1991, Ballard and Fullerton 1992, and Sandmo 1998). Much of this literature focuses on the marginal cost of public funds (MCF), which is defined as the multiplier to be applied to the net government revenue required to finance the additional provision of a public good in order to calculate its true marginal social cost. Pigou (1947) suggested that the MCF would necessarily be greater than one when revenue was raised by distortionary taxation, but it was subsequently recognised that this intuition is not generally correct. Even when the distribution of welfare among households is not an issue, and the revenue required to finance the additional public good provision comes from increases in distortionary taxes, it is possible for income effects on taxed consumption goods to cause the MCF to be less than one, as Atkinson and Stern (1974) showed. When the distribution of household welfare matters, and redistribution takes place through a linear income tax which has both distortionary and non-distortionary elements (the positive marginal income tax rate and the uniform lump-sum transfer respectively), it is also possible for the MCF to be less than one, as Sandmo (1998) showed.

Because the MCF can therefore be greater than, equal to, or less than one, it is well understood that care is required in computing the marginal social cost of public good provision with distortionary taxation. However, most of the existing literature accepts that this cost is correctly measured by multiplying the actual revenue required (net of any induced effects caused by changes in the demands for taxed goods resulting from the additional public good provision) by the MCF. In this paper I argue that this measure of the marginal social cost of public good provision is correct only in special cases. As Drèze and Stern (1990) showed, to express the marginal social cost of a public project in terms of revenue, it is in general necessary to evaluate the net effect of such a project on shadow, rather than actual, government revenue. This point has implications which do not appear to be widely appreciated in the context of cost-benefit rules for public good provision. The marginal social cost of public good provision is correctly measured in the general case by the net effect of such provision on shadow government revenue. The marginal social cost so defined can be compared directly with the marginal social benefit of the public good (given by an appropriately-weighted sum of households' marginal willingness to pay) in order to determine whether additional provision is justified, and thus there is no need to use the concept of the MCF to make decisions about public good provision with distortionary taxation. That is, in formulating cost-benefit rules for public good provision with distortionary taxation, it is necessary to take on board Drèze and Stern's general point that there is no need in cost-benefit analysis for a separate concept of the shadow price of government revenue.

Section 2 of the paper sets out a model for analysing the appropriate cost-benefit rule for public good provision, which is a simplified version of that in Drèze and Stern (1990). Section 3 shows that a generally valid rule for public good provision involves a comparison of the marginal social benefit of the public good with the overall effect of its provision on shadow government revenue. Section 4 shows that, in special cases, this general rule is equivalent to a rule for public good provision in which the marginal social cost of a public good is given by its net effect on actual government revenue multiplied by the MCF. Section 5 shows that, if the economy trades goods at given world prices, there are circumstances in which the net effect of public good provision is correctly measured by effects on actual government revenue, without any adjustment by the MCF. Provided that the additional public good supply does not affect demands for non-traded goods, this means that a measure of the marginal social cost of the public good exists which is straightforward to implement and does not require any assumption that policy has been optimised. In general, however, demands for non-traded goods will be affected by the level of public good provision, and this complicates the measurement of the marginal social cost of the public good, as does the possibility that some goods are not traded at given world prices. I argue that the evaluation of the net effect of public good provision on the shadow revenue raised from non-traded goods and goods not traded at given world prices is a more significant practical problem than estimating the MCF when measuring the marginal social cost of a public good in economies using distortionary taxation. There is a brief conclusion to the paper in section 6.

2 The model

The model used in this paper is a simplified version of that in Drèze and Stern (1990). There are N private goods, which are either non-traded or traded at given world prices, and a single pure public good G. The vector of world prices for traded goods, in foreign exchange, is $\mathbf{p}^w$: this vector has zero components corresponding to the non-traded goods. The vector of net imports is $\mathbf{n}$: positive components correspond to imported goods, negative components to exported goods, and components corresponding to non-traded goods are identically zero. Private production is carried out by a single competitive firm which trades at producer prices $\mathbf{p}$ and has a strictly convex production set. The private firm's profit function is $\pi(\mathbf{p})$ and its (vector-valued) net supply function is $\mathbf{y}(\mathbf{p})$. Goods which the firm produces as outputs appear as positive components of its net supply vector, while those used as inputs are negative components of this vector. The vector of public sector net supply of private goods is $\mathbf{z}$, positive and negative components of which correspond respectively to outputs and inputs. The pure public good is produced by the public sector using private goods as inputs.

There are H households, each of which trades at consumer prices $\mathbf{q}$ and has lump-sum income $m^h = (1-\tau)\theta^h\pi(\mathbf{p}) + b$, where τ denotes the tax rate on the private firm's profits, θ^h denotes the share of household h in the firm's profits, and b denotes a uniform transfer paid to each household by the tax authority. Each household chooses a utility-maximising vector of net demands for private goods subject to its budget constraint and the given quantity of the public good. The resulting (vector-valued) net demand functions are $\mathbf{x}^h(\mathbf{q}, m^h, G)$: positive components of these vectors correspond to goods demanded, and negative components to goods supplied. Household preferences are represented by the indirect utility functions $v^h(\mathbf{q}, m^h, G)$. The aggregate net demand function is $\mathbf{x}(\mathbf{q}, m^1, ..., m^H, G) = \sum_1^H \mathbf{x}^h(\mathbf{q}, m^h, G)$.

The vector of indirect taxes on net demands is $\mathbf{t}$.[1] The sign of the product $t_i x_i$ shows whether there is a tax or a subsidy on the net demand for a particular good, so that a tax is indicated by $t_i > 0$ if $x_i > 0$, and by $t_i < 0$ if $x_i < 0$. The vector of trade taxes on net imports is $\mathbf{f}$. This vector has zero components corresponding to non-traded goods, and there are similar sign conventions to those for indirect taxes, so that $f_i > 0$ corresponds to a tariff for a good which is imported, and to an export subsidy for a good which is exported. Hence the consumer price of a traded good is $q_i = ep_i^w + f_i + t_i$ and the producer price of such a good is $p_i = ep_i^w + f_i$, where e is the market exchange rate, i.e. the domestic currency price of a unit of foreign exchange. For a non-traded good, the relationship between consumer and producer prices is given by $q_i = p_i + t_i$.

The economy has a given endowment of foreign exchange, F, which is owned by the government.[2] An equilibrium in this economy requires that goods markets clear and the balance of payments balances:

$$\mathbf{x}(\mathbf{q}, m^1, ..., m^H, G) = \mathbf{y}(\mathbf{p}) + \mathbf{z} + \mathbf{n} \tag{1}$$

$$\mathbf{p}^w.\mathbf{n} = F \tag{2}$$

Assuming (without loss of generality) that the public sector trades at producer prices, the government budget constraint can be written as:

$$\mathbf{t}.\mathbf{x} + \mathbf{f}.\mathbf{n} + \mathbf{p}.\mathbf{z} + \tau\pi - \sum_{h=1}^{H} b + eF = 0 \tag{3}$$

[1] Note that linear taxes on goods supplied by households are included in $\mathbf{t}$.

[2] The endowment of foreign exchange plays no essential role in the analysis, and its value can be set equal to zero if desired.

It is straightforward to show, using the budget constraint of households in the aggregate, that the equilibrium conditions (1) and (2) imply that the government budget constraint (3) is satisfied.

Public policy in this economy is implemented by a planner who chooses some control variables to maximise a Bergson-Samuelson social welfare function $W(..., v^h(\mathbf{q}, m^h, G), ...)$ subject to the constraints given by the equilibrium conditions (1) and (2). Since there are $N+1$ constraints, the planner requires at least $N+1$ control variables to solve this problem. In the case where there are only $N+1$ control variables, the planner has no degrees of freedom in policy choice: the values of the control variables are 'fully determined' (in Drèze and Stern's phrase) by the equilibrium conditions. This fully determined case in which the planner does no genuine policy optimisation plays an important role in the following analysis. However, the general framework also applies to cases in which there are more than $N+1$ control variables, so that it is possible for the planner to undertake some genuine optimisation.

The Lagrangean for the planner's social welfare maximisation problem can be written as:

$$L(.) = W(..., v^h(\mathbf{q}, m^h, G), ...) - \mathbf{s}.\left[\mathbf{x}(\mathbf{q}, m^1, ..., m^H, G) - \mathbf{y}(\mathbf{p}) - \mathbf{z} - \mathbf{n}\right] - \mu\left[\mathbf{p}^w.\mathbf{n} - F\right] \qquad (4)$$

In (4), $\mathbf{s}$ denotes the vector of Lagrange multipliers on the goods market equilibrium constraints and μ denotes the Lagrange multiplier on the foreign exchange constraint. The control variables used to solve the planner's problem are drawn from the following: $\mathbf{t}, \mathbf{f}, \mathbf{n}$, and b: the only assumption made about these control variables is that the planner can use at least $N+1$ of them. A number of potential policy variables are taken as parameters of the planner's problem: these include $\mathbf{z}, G$, and τ. It is assumed that a solution to the planner's problem always exists, and that this solution always involves the use of some distortionary taxes.[3] Let $W^*(\mathbf{z}, G, F)$ be the maximum social welfare resulting from a solution to the planner's problem at given values of $\mathbf{z}, G$, and F. By the envelope theorem, the effect of a small change in a particular z_i on maximised social

[3] In particular, this assumption means that whatever revenue is raised from taxing profits (a lump-sum tax in this model) is insufficient to satisfy the government budget constraint.

welfare is given by the derivative of the Lagrangean (4) with respect to z_i, holding the control variables and the Lagrange multipliers fixed at the values which solve the maximisation problem. Hence the values of the Lagrange multipliers $\mathbf{s}$ give the effect on maximised social welfare of a small change in z_i, and are thus the shadow prices of the N goods. Similarly, the value of μ is the shadow price of foreign exchange.

3 A general rule for public good provision

One of the parameters in the planner's social welfare maximisation problem is G. A small increase dG in the supply of the public good is desirable if it increases maximised social welfare. To evaluate the effect of dG on maximised social welfare it is convenient to rewrite the Lagrangean (4) as follows. Since, from the aggregate budget constraint of households, $\tau\pi = \mathbf{p.y} - \mathbf{q.x} + \sum_h b$, the constraint terms in the Lagrangean can be written as:

$$\begin{aligned} & \tau\pi - \left(\mathbf{p.y} - \mathbf{q.x} + \sum_h b\right) - \mathbf{s}.\left[\mathbf{x} - \mathbf{y} - \mathbf{z} - \mathbf{n}\right] - \mu\left[\mathbf{p}^w.\mathbf{n} - F\right] \\ = \quad & (\mathbf{q} - \mathbf{s}).\mathbf{x} + (\mathbf{s} - \mathbf{p}).\mathbf{y} + (\mathbf{s} - \mu\mathbf{p}^w).\mathbf{n} + \mathbf{s.z} + \tau\pi - \sum_h b + \mu F \end{aligned} \tag{5}$$

Expression (5) can be interpreted as follows. The components of the vector $\mathbf{q} - \mathbf{s}$ give the difference between consumer and shadow prices of goods, or shadow consumer taxes. Similarly, the components of the vector $\mathbf{s} - \mathbf{p}$ are shadow producer taxes, and those of the vector $\mathbf{s} - \mu\mathbf{p}^w$ are shadow trade taxes. With these interpretations, and also assuming that the public sector trades $\mathbf{z}$ and F at shadow prices, (5) gives shadow government (net) revenue. Thus the Lagrangean for the planner's problem can be written as:

$$L(.) = W(..., v^h(\mathbf{q}, m^h, G), ...) + (\mathbf{q} - \mathbf{s}).\mathbf{x} + (\mathbf{s} - \mathbf{p}).\mathbf{y} + (\mathbf{s} - \mu\mathbf{p}^w).\mathbf{n} + \mathbf{s.z} + \tau\pi - \sum_h b + \mu F \tag{6}$$

Assume that the planner's problem has been solved, and consider the effect of an increase dG on maximum social welfare, which is now written solely as a function of G, $W^*(G)$. The change in social welfare is $dW^* = (\partial W^*/\partial G)\, dG$, and the increase in G should therefore be implemented if $\partial W^*/\partial G > 0$. By the envelope theorem, $\partial W^*/\partial G = \partial L/\partial G$, where L is given by (6). Hence

$$\frac{\partial W^*}{\partial G} = \sum_{h=1}^{H} \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial G} + \sum_{j=1}^{N} (q_j - s_j) \frac{\partial x_j}{\partial G} + \sum_{j=1}^{N} s_j \frac{\partial z_j}{\partial G} \tag{7}$$

where $\partial z_j/\partial G$ denotes the change in public sector input j which is required to produce an increase in G, and it is assumed that $\partial z_j/\partial G \leq 0$, $j = 1, ..., N$. Since $\partial v^h/\partial G = \left(\partial v^h/\partial m^h\right) q_G^h$, where q_G^h denotes household h's marginal willingness to pay for the public good, the condition for a small increase in the public good to raise social welfare can be written as

$$\sum_{h=1}^{H} \beta^h q_G^h > -\sum_{j=1}^{N} s_j \frac{\partial z_j}{\partial G} - \sum_{j=1}^{N} (q_j - s_j) \frac{\partial x_j}{\partial G} \tag{8}$$

where $\beta^h \equiv \left(\partial W/\partial v^h\right) \left(\partial v^h/\partial m^h\right)$ is the social marginal utility of income, or welfare weight, of household h.

Expression (8) gives a general rule for deciding whether a project involving a small increase in the supply of the public good should be accepted. If the welfare-weighted sum of households' marginal willingness to pay for the public good exceeds the cost of the project at shadow prices less the effect that the project has on shadow consumer tax revenue because of complementarity or substitutability between the public good and household demands for private goods, then the project should be undertaken. This rule is completely general: it applies both to the fully determined case, in which the planner has no degrees of freedom in policy choice, and to cases where the planner is able to do some genuine policy optimisation. The rule does not require any assumptions about constant returns to scale or full taxation of profits from private sector production. To decide whether to increase public good provision, it is necessary to estimate the welfare-weighted direct effect of such provision, in terms of households' marginal willingness to pay, and the overall effect of the increased provision on shadow government revenue. Only the latter effect is unfamiliar, and so the remainder of the paper concentrates on it.

4 The general rule and the marginal cost of public funds

The general cost-benefit rule for public good provision represented by expression (8) does not explicitly involve the marginal cost of public funds (MCF). In order to understand the relationship between this general rule and cost-benefit rules which do explicitly involve the MCF, two special cases of the general rule are discussed in detail in this section. There is also a brief consideration of whether a useful general definition of the MCF exists.

4.1 The Atkinson-Stern rule

Suppose that all private goods are non-traded, and that all H households are identical. Suppose also that, under a suitable normalisation rule, shadow prices of private goods, $\mathbf{s}$, are equal to their producer prices $\mathbf{p}$.[4] Under these assumptions, the social welfare function becomes $Hv^h(\mathbf{q}, b, G)$, and shadow consumer taxes $\mathbf{q} - \mathbf{s}$ coincide with actual consumer taxes $\mathbf{t}$. Hence expression (8) becomes

$$\begin{aligned} H\frac{\partial v^h}{\partial m^h}q_G^h &> -\sum_{j=1}^{N} p_j \frac{\partial z_j}{\partial G} - \sum_{j=1}^{N} t_j \frac{\partial x_j}{\partial G} \\ \text{or} \qquad Hq_G^h &> \frac{1}{\partial v^h/\partial m^h}\left(-\sum_{j=1}^{N} p_j \frac{\partial z_j}{\partial G} - \sum_{j=1}^{N} t_j \frac{\partial x_j}{\partial G}\right) \end{aligned} \tag{9}$$

According to (9), in this special case a small increase in the public good raises social welfare if the sum of the identical households' marginal willingness to pay for the increase exceeds the net actual revenue cost of the increase adjusted by a factor $1/\left(\partial v^h/\partial m^h\right)$. This adjustment factor is required for public sector revenue to be comparable with the identical households' marginal willingness to pay when making social welfare calculations, and it corresponds to the MCF as defined by Atkinson and Stern (1974).

[4] There are two sets of sufficient conditions for shadow and producer prices to coincide. One is that there should be constant returns to scale in production, only one non-produced input, and no joint production. The other is that all final consumption goods can be taxed, the taxes on these goods should be set optimally, and private sector production should involve either constant or decreasing returns to scale, with any private sector profits being fully taxed.

To see this, note that any solution to the planner's problem is assumed to involve the use of some distortionary taxes. In an economy of identical households, private sector profits should be fully taxed, so the use of distortionary taxes in addition to taxes on profits implies that the constraint that private profits cannot be taxed at a rate above 100% binds. Consequently, for each identical household, the following condition holds:

$$\frac{\partial v^h}{\partial m^h} - \sum_{j=1}^{N} s_j \frac{\partial x_j^h}{\partial m^h} < 0$$

and since shadow prices equal producer prices

$$\frac{\partial v^h}{\partial m^h} - \sum_{j=1}^{N} p_j \frac{\partial x_j^h}{\partial m^h} < 0 \tag{10}$$

Each identical household has the budget constraint $\mathbf{q}.\mathbf{x}^h = b$, and thus has demand functions which satisfy

$$\sum_{j=1}^{N} (p_j + t_j) \frac{\partial x_j^h}{\partial m^h} = 1$$

so that (10) can be written

$$\frac{\partial v^h}{\partial m^h} + \sum_{j=1}^{N} t_j \frac{\partial x_j^h}{\partial m^h} < 1 \tag{11}$$

From (11), a sufficient condition for $1/\left(\partial v^h/\partial m^h\right) > 1$ is that $\sum_{j=1}^{N} t_j \left(\partial x_j^h/\partial m^h\right) > 0$. This condition is exactly the sufficient condition Atkinson and Stern derive for the MCF to be greater than one. However, the term $\sum_{j=1}^{N} t_j \left(\partial x_j^h/\partial m^h\right)$ cannot be unambiguously signed,[5] so that there is no presumption that the adjustment factor which has to be applied to the net actual revenue cost of the extra public good in (9) is always greater than one. To understand the intuition behind this result, it is important to recognise that the increased taxation required to finance the additional public good has an income effect on household net demand functions, and thus on actual tax revenue. If $\sum_{j=1}^{N} t_j \left(\partial x_j^h/\partial m^h\right) > 0$, then taxed goods are normal on average (weighted by tax

[5] A sufficient condition for this term to be positive is that all taxed goods are normal.

rates), so that this income effect lowers actual revenue at given tax rates. Since the substitution effects of taxation impose efficiency costs, the income effect in this case results in the MCF being greater than one. However, if $\sum_{j=1}^{N} t_j \left(\partial x_j^h / \partial m^h\right) < 0$, then the income effect raises actual revenue at given tax rates. If this term is sufficiently negative, so that taxed goods are sufficiently inferior on average, the income effect is strong enough to outweigh the substitution effects which impose efficiency costs, and hence the MCF is less than one.

4.2 The Sandmo rule

The special case of the previous sub-section abstracts from differences between households in order to show that, even when efficiency is the sole consideration, the MCF is not necessarily greater than one when revenue is raised by distortionary taxation. But the main reason why distortionary taxes exist is that distribution matters, so that practical cost-benefit rules for public good provision must take account of distributional objectives. Suppose, therefore, that the special case of the previous sub-section is modified to allow for differences among the H households: in particular, let households differ in the amount of effective labour they can supply by working for one hour. Furthermore, suppose that the only tax is a constant marginal tax rate t on labour income, which finances the uniform transfer b and the public good. The other assumptions of the previous sub-section are maintained: all private goods are non-traded, and the shadow prices of private goods equal producer prices.

Let good l be labour, and let w^h be the hourly pre-tax wage rate of household h, which is the producer price (and hence the shadow price) of an hour's labour supplied by household h.[6] The consumer price of an hour's labour supplied by household h is $q^h = (1-t)w^h$, and thus $q^h - s^h = -tw^h$. The general expression (8) thus becomes

$$\sum_{h=1}^{H} \beta^h q_G^h > -\sum_{j=1}^{N} p_j \frac{\partial z_j}{\partial G} + t \sum_{h=1}^{H} w^h \frac{\partial x_l^h}{\partial G} \tag{12}$$

[6] If household h supplies h units of effective labour in one hour, $w^h = hp_l$, where p_l is the constant producer price of a unit of effective labour.

Let $\beta = (1/H)\sum_{h=1}^{H}\beta^h$ be the average social marginal utility of income. Using this definition, (12) can be written as

$$\frac{H\sum_{h=1}^{H}\beta^h q_G^h}{\sum_{h=1}^{H}\beta^h} > \frac{1}{\beta}\left(-\sum_{j=1}^{N} p_j \frac{\partial z_j}{\partial G} + t\sum_{h=1}^{H} w^h \frac{\partial x_l^h}{\partial G}\right) \tag{13}$$

Expression (13) is a slight generalisation of the condition given by Sandmo (1998) for public good provision in a model where households differ in their ability to supply effective labour and there is a linear income tax.[7] In this special case a small increase in the public good raises social welfare if H times the marginal willingness to pay of a socially representative consumer, defined by using social marginal utilities of income as weights, exceeds the net actual revenue cost of the increase adjusted by a factor $1/\beta$. This adjustment factor corresponds to Sandmo's definition of the MCF.

Sandmo shows that, in certain circumstances, his definition of the MCF will be strictly less than one. To see this in the context of (13), note that, from (6), the Lagrangean for the planner's problem can be written as $L(.) = W(.) + R_{\mathbf{s}}$, where $R_{\mathbf{s}}$ denotes shadow revenue. Hence if the uniform lump-sum transfer b is a control variable for the planner's problem, the first-order condition $\partial W/\partial b + \partial R_{\mathbf{s}}/\partial b = 0$ must hold.[8] In the special case under consideration, this first-order condition is

$$\sum_{h=1}^{H}\beta^h - t\sum_{h=1}^{H} w^h \frac{\partial x_l^h}{\partial m^h} - H = 0$$

Dividing through by H and using the definition of β, this first-order condition can be written as

$$\beta = 1 + \frac{t}{H}\sum_{h=1}^{H} w^h \frac{\partial x_l^h}{\partial m^h} \tag{14}$$

If leisure is a normal good, $\partial x_l^h/\partial m^h > 0$, and hence $\beta > 1$, so that $1/\beta$, the MCF, is less than one. The intuition behind this result is as follows. When additional public good provision is financed

[7] The relevant condition is equation (15) in Sandmo's paper, which characterises optimal public good provision.

[8] This does not imply that the planner is necessarily able to undertake genuine policy optimisation with respect to b: it is also consistent with the case in which the values of the control variables are fully determined by the equilibrium conditions. The interpretation of this second case is that government budget balance is maintained when there is an increase in public good provision by a reduction in the uniform lump-sum transfer.

by a reduction in the uniform lump-sum transfer, the MCF will be less than one if the reduction in the transfer has a positive income effect on labour supply, because of the negative substitution effect on labour supply due to the marginal income tax rate.

Although (13) is a cost-benefit rule for public good provision in which the marginal social cost of such provision is expressed as the effect on actual net revenue adjusted by a suitably-defined MCF, it is not clear that the rule given by (13) offers any advantages over the rule given by (12), which is simply the general rule (8) adapted to the special case of this sub-section. In (12) the marginal social cost of the public good is given by the net effect on actual revenue, with no adjustment by the MCF, and this is compared to the sum of households' welfare-weighted marginal willingness to pay. In (13) the marginal social cost of the public good is given by the net effect on actual revenue adjusted by the MCF, and this is compared to H times the marginal willingness to pay of a socially representative consumer. However, the definition of the socially representative consumer in (13) requires knowledge of all households' welfare-weighted marginal willingness to pay, so the use of the MCF as part of the cost measure in (13) does not lead to any offsetting reduction in the information required for the benefit measure in comparison with (12).

4.3 The marginal cost of public funds in the general case

Both the Atkinson-Stern rule and the Sandmo rule for public good provision take the form of a comparison of some overall measure of marginal willingness to pay for the public good with an adjusted measure of the net actual revenue cost of the additional public good. The adjustment factor is the MCF, and can be either greater or less than one. However, both the rules depend on the assumption that shadow prices are, under a suitable normalisation, equal to producer prices. If this assumption does not hold, and, as noted above, the conditions under which shadow and producer prices coincide are stringent, the revenue effects which have to be considered when deciding whether to increase the provision of a public good are those on shadow, not actual, revenue. It is clear that, in the general case, the net shadow revenue cost of public good provision can always be expressed as the net actual revenue cost adjusted by a suitably-defined MCF, namely the net change in shadow revenue due to the additional public good provision divided by the net change in actual revenue due to such provision. However, this definition of the MCF requires knowledge of the cost of provision at shadow prices less the effect that provision has on shadow consumer tax revenue. Consequently there is nothing to be gained by measuring the marginal social cost of public good provision in this way rather than as the net effect of public good provision on shadow revenue, as in the general rule (8).

5 Implementing the general rule

The previous section discussed the relationship between the general rule for public good provision given by (8) and rules involving the MCF, and argued that rules involving the MCF did not have any advantages over the general rule. This section considers the problems of implementing the general rule in practice.

Suppose, as in section 2, that $T(< N)$ of the private goods are traded at given world prices $\mathbf{p}^w$. Consider the fully determined case in which the planner has only $N+1$ control variables with which to solve the social welfare maximisation problem. Let T of these control variables be the net imports of traded goods n_t, $t = 1, ..., T$, and let the remaining $N+1-T$ control variables be a subset of the producer prices. In this case, the values of the control variables are determined by the $N+1$ equilibrium conditions (1) and (2), while the first order conditions obtained by differentiating the Lagrangean (4) with respect to the control variables determine the values of the Lagrange multipliers, which are equal to the shadow prices of private goods and foreign exchange.

The first-order conditions for the net imports of traded goods are

$$s_t - \mu p_t^w = 0 \qquad t = 1, ..., T$$

and thus the shadow prices of traded private goods are equal to their world prices multiplied by the shadow price of foreign exchange. Hence the general rule (8) becomes

$$\sum_{h=1}^{H} \beta^h q_G^h > -\sum_{t=1}^{T} \mu p_t^w \frac{\partial z_t}{\partial G} - \sum_{k=T+1}^{N} s_k \frac{\partial z_k}{\partial G} - \sum_{t=1}^{T} (q_t - \mu p_t^w) \frac{\partial x_t}{\partial G} - \sum_{k=T+1}^{N} (q_k - s_k) \frac{\partial x_k}{\partial G} \tag{15}$$

where $t = 1, ..., T$ denotes traded goods and $k = T+1, ..., N$ denotes non-traded goods.

Suppose first that $\partial x_k / \partial G = \partial z_k / \partial G = 0$, $k = T+1, ..., N$, so that a small increase in the public good has no effects on household net demands for non-traded goods or public sector demands for inputs of non-traded goods. Since, for traded goods, $q_t = ep_t^w + f_t + t_t$, (15) can then be written as

$$\sum_{h=1}^{H} \beta^h q_G^h > -\sum_{t=1}^{T} \mu p_t^w \frac{\partial z_t}{\partial G} - \sum_{t=1}^{T} [f_t + t_t + (e - \mu) p_t^w] \frac{\partial x_t}{\partial G} \tag{16}$$

If the market exchange rate e equals the shadow exchange rate μ, (16) becomes

$$\sum_{h=1}^{H} \beta^h q_G^h > -\sum_{t=1}^{T} ep_t^w \frac{\partial z_t}{\partial G} - \sum_{t=1}^{T} (f_t + t_t) \frac{\partial x_t}{\partial G} \tag{17}$$

This states that a small increase in the public good increases social welfare if the welfare-weighted sum of households' marginal willingness to pay for the public good exceeds the direct cost of the public good computed using the domestic currency value of world prices less any induced effects on actual consumer and trade tax revenue resulting from complementarity or substitutability between the public good and household demands for private goods.

If the market and shadow exchange rates differ, then, from (16), the rule for public good provision is a little more complex. The direct cost of the public good must now be valued at world prices adjusted by the shadow value of foreign exchange, and the effects resulting from complementarity or substitutability between the public good and household demands for private goods must now include the value at world prices of the induced changes in household demands adjusted for the difference between the market and the shadow exchange rates.

Expressions (16) and (17) show that it is possible to obtain applicable rules for public good provision without having to make any assumptions about genuine policy optimisation on the part of the planner or strong assumptions about the relationship between shadow and producer prices. The assumptions required to derive (16) and (17) are, first, that the economy trades private goods at given world prices; second, that an equilibrium exists for any given values of indirect taxes, trade taxes, the uniform transfer to households, the profits tax on private sector producers, public sector net supplies of private goods, and the public good; and third, that a small change in the supply of the public good has no effects on household net demands for non-traded goods or public sector demands for inputs of non-traded goods. The most restrictive of these assumptions is the third one, which will be discussed shortly. Before doing so, however, it is worth emphasising that the assumption that the world prices of traded goods are exogenously given is a reasonable one for many economies, and that (16) and (17) do not require that shadow prices equal producer prices, so that there is no need to make any assumptions about the optimal taxation of all final consumption goods and the absence of profits in household budget constraints. It is also worth pointing out that there is no need for any concept of the marginal cost of public funds in these rules: although effects on actual revenue enter into the rules, they do so without any adjustment.

The assumption that small changes in the supply of the public good have no effects on household net demands for non-traded goods or public sector demands for inputs of non-traded goods, on which (16) and (17) depend, is clearly implausible, and relaxing it leads to a more complicated rule for public good provision. From (15) it is clear that the direct cost of the public good must now include the terms $-\sum_{k=T+1}^{N} s_k \partial z_k / \partial G$, which give the cost, at shadow prices, of the non-traded inputs required to increase the production of the public good. Furthermore, the induced effects subtracted from the direct cost must include the terms $\sum_{k=T+1}^{N} (q_k - s_k) \partial x_k / \partial G$, which give the effects on shadow consumer tax revenue due to changes in household demands for non-traded goods resulting from the increase in the supply of the public good. Although there are special cases in which these additional terms correspond to changes in actual government revenue, these are of limited use in practice, since they require the assumption of a great deal of policy optimisation by the planner.[9] The practical application of these rules will typically require that the shadow prices of non-traded goods be estimated, and used to compute the cost of the increased non-traded inputs and the effects on shadow consumer tax revenue from non-traded goods. The estimation of shadow prices for non-traded goods is not, in general, straightforward, but neither is it impossible, so this general rule for public good provision can be used even when the increase in the public good has implications for non-traded inputs and demand for non-traded goods.[10]

One final complication which must be mentioned concerns the possibility that some private goods are not traded at given prices. If the economy is able to influence world prices for some traded goods by the amounts it trades, then the shadow prices of these traded goods will differ from world prices, and they will not, in general, be given by the marginal cost of an import or the marginal revenue from an export. As with the shadow prices of non-traded goods, the shadow prices of these traded goods will have to be estimated in a way which is neither wholly straightforward nor impossible.

The analysis in this section has shown that generally valid and applicable rules for public good provision do exist under reasonable conditions, and that, to some extent, these rules can be expressed in terms of effects on actual government revenue. Provided that the economy trades private goods at given world prices, the shadow prices for traded goods can be related to world market prices without making any assumptions about genuine policy optimisation, and hence the effects on shadow revenue arising from changes in traded goods can be expressed in terms of actual consumer and trade taxes. Although matters are less straightforward as far as non-traded

[9] For example, if all consumer and trade taxes are set optimally, and there are no profits in household budget constraints, then the shadow prices of all private goods, traded and non-traded, are equal to producer prices, so that these extra terms correspond to the cost of the increased non-traded inputs required and the change in actual consumer tax revenue from non-traded goods.

[10] A large literature exists on the estimation of shadow prices for non-traded goods. Little and Mirrlees (1990) contains a short account of the main issues and difficulties.

goods, or goods which are not traded at given world prices, are concerned, this does not alter the conclusion that applicable general rules for public good provision exist. Furthermore, these rules do not require the concept of the marginal cost of public funds. The lesson to be drawn is that the important practical problems involved in implementing a generally valid measure of the marginal social cost of public good provision concern the estimation of shadow prices for non-traded goods and goods which are not traded at exogenous world prices, rather than the estimation of the marginal cost of public funds.

6 Conclusion

This paper has shown that a comparison of the appropriately-weighted sum of households' marginal willingness to pay for a public good with the net effect of the marginal increase in the supply of the public good on shadow government revenue is a generally valid cost-benefit rule for public good provision in economies with distortionary taxation. This rule does not depend on any assumption that existing policy is optimal. Furthermore, if the economy trades goods at given world prices, much of the measurement of the net effect of increased public good provision on shadow government revenue can be done straightforwardly in terms of effects on actual government revenue. The practical problems in measuring the net effect on shadow revenue involve the need to estimate shadow prices of non-traded goods and goods which are not traded at given world prices. This implies that, in order to measure the marginal social cost of a public good in practice, estimates of these shadow prices are required, rather than estimates of the marginal cost of public funds. The latter, as this paper has shown, is neither required to measure the marginal social cost of a public good nor generally valid as a component of this cost.

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